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227. Proposed by O. W. ANTHONY, Head of Mathematical Department, DeWitt Clinton High School, New York City.

Construct a parallelogram having given a side and the distances of its vertices from a given point.

I. Solution by M. E. GRABER, A. M., Heidelberg University, Tiffin, Ohio.

Let a be the side of the parallelogram; b and c the distances of the given point from the extremities of a; and d and e the distances of the given point from the side equal and parallel to a, say a'. Construct triangle abc, considering a the base, and erect a perpendicular p, from the vertex to a. Construct another triangle with d and e as sides and a line equal to a as base and determine its altitude l. Then from vertex of triangle abc measure off l on p* and through the extremity of l draw a parallel to a and with d and e as radii and the vertex of triangle abc as a center, describe arcs cutting this parallel.† The intercepted parallel is a'. From a and a' the parallelogram is easily constructed for two equal parallel sides determine a parallelogram, and at the same time the distances from the vertices to the given point are b, c, d, and e as specified in the problem.

II. Solution by L. E. NEWCOMB, Los Gatos, Cal.

(1) Let A be the given point; a, b, c, d the given lines from the vertices and n the given side. Construct with sides a, b, n and c, d, n the triangles AA_1A_2 and AA_3A_4 respectively. From A draw AB, AC perpendicular to A_1A_2 and A_3A_4 respectively.

If A lies between the given sides or their extension, prolong CA to B', making AB' equal to AB; from B' draw $B'A_2'$ parallel to A_3A_4 and make $B'A_2$ equal to BA_2; extend $A_2'B'$ to A_1' making A_2A_1' equal to n. Join $A_1'A_4$, $A_2'A_3$, AA_1' and AA_2'.

Since $A_1'A_2'$ and A_3A_4 are by construction each equal to n and parallel to each other and since the lines AA_1', $A_2'A$ are equal, respectively, to AA_1, AA_2, and AA_1', AA_2', AA_3, AA_4 are respectively equal to a, b, c, d, it follows that $A_1'A_2'A_3A_4$ is the required parallelogram.

(2) The case when A lies on the side corresponding to A_1A_2 is easily disposed of, since AB' must $=0$ and $A_1A_2=a+b$.

(3) If A lies without the given sides: from A lay off on AC AB'' equal to AB; draw $B''A_2''$ equal to BA_2 and parallel to A_4A_3; extend $A_2''B_2''$ to A_1'' making $A_2''A_1''$ equal to n; join $A_1''A_4$, $A_2''A_3$, AA_1'' and AA_2''; $A_1''A_2''A_3A_4$ is the parallelogram required, the proof being similar to that in case (1).

It is taken for granted in the above solutions that the length of each line from A to each vertex is given; otherwise there may be constructed 12 parallelograms each meeting the conditions.

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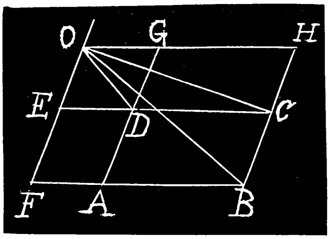
*There arise two possibilities according as l is laid off in the same direction as p or in the opposite direction. ED.

†Again two possibilities arise for we may take the segment on either side of the perpendicular. ED.

III. **Solution by G. B. M. ZERR, A. M., Ph. D., Parsons, W. Va.**

Let O be the given point, AB the given side. Let $OA=a$, $OB=b$, $OC=c$, $OD=d$, $AB=e$, $FA=x$, $AD=z$, $DG=y$, $\angle OFA=\theta$.



Then $a^2=(y+z)^2+x^2-2x(y+z)\cos\theta$.....(1),

$b^2=(y+z)^2+(x+e)^2-2(x+e)(y+z)\cos\theta$..........(2),

$c^2=y^2+(x+e)^2-2y(x+e)\cos\theta$.........(3),

$d^2=y^2+x^2-2xy\cos\theta$......... (4).

From (1) and (2),

$y+z=x\cos\theta\pm\sqrt{(a^2-x^2\sin^2\theta)}=(x+e)\cos\theta$

$\pm\sqrt{[b^2-(x+e)^2\sin^2\theta]}$..........(5).

From (3) and (4),

$y=(x+e)\cos\theta\pm\sqrt{[c^2-(x+e)^2\sin^2\theta]}=x\cos\theta\pm\sqrt{(d^2-x^2\sin^2\theta)}$......... (6).

Let $4b^2e^2=A^2$, $(a^2-e^2-b^2)=B$, $2ce=C$, $d^2-e^2-c^2=D$.

Then (5) and (6) become after reduction

$$A^2-B^2=[4Be(e+x)+A^2+4e^2(e+x)^2]\sin^2\theta \text{..........(7)},$$

$$C^2-D^2=[4De(e+x)+C^2+4e^2(e+x)^2]\sin^2\theta \text{.........(8)}.$$

Eliminating $\sin\theta$ we get

$$4e^2(A^2-B^2-C^2+D^2)(e+x)^2+4e[D(A^2-B^2)$$
$$-B(C^2-D^2)](e+x)=B^2C^2-A^2D^2 \text{.........(9)}.$$

Equation (9) determines x. Then (7) or (8) determines θ, (3) or (4) determines y, (1) or (2) determines z, and the parallelogram is determined in all respects.

228. **Proposed by O.E. GLENN, A.M., Fellow in Mathematics, University of Pennsylvania, Philadelphia, Pa.**

Given a point O without a circle S; two arbitrary lines through O cut S in the points A, A', and B, B', respectively. Prove, by pure geometry, that the four circles through OAR, OBR, $OA'R'$, $OB'R'$, respectively, intersect in points collinear with O; R and R' being points upon S arbitrarily chosen.

No solution has been received.

CALCULUS.

179. **Proposed by B. F. FINKEL, A. M., Drury College, Springfield, Mo.**

Discuss the integrals of the equation $x(1-x)w''+[1-(a+b+1)x]w'-abw=0$ in the vicinities $x=0$, and $x=1$; indicating the form for the latter vicinity when $a+b=1$. Also when $1-a-b$ is an integer k. [From Forsyth's *Linear Differential Equations*, Ex. 6, p. 103].